MATH 207A MIDTERM 2

INTRO TO DIFFERENTIAL EQUATIONS

JULY 31, 2024

NAME: _____

Exam Guidelines.

- Write your name above.
- Remain on this title sheet until you are instructed to begin the exam.
- The time given for this exam is 60 minutes.
- There are 5 questions and each question is worth 10 points.
- You may not use any electronic device except for a non-graphing, non-programmable calculator (such as a TI30 or equivalent). A calculator is not necessary for this exam.
- For full credit, show your work for each problem. You may use the space directly below the problem statement and the blank page following each problem.
- Clearly mark your final answer. For questions with multiple parts, clearly mark which part you are working on.
- Let me know if you have a question about what a problem is asking. If you are taking this exam at a proctoring center, the proctor can direct questions to seanhr@uw.edu.

Good luck!

Problem 1 [10 points]. Find the general solution to the following differential equations.

(a) [5 points] $y'' - 4y' + 4y = 0$.

(b) [5 points] $y'' - 4y' + 5y = 0$.

Problem 2 [10 points]. Solve the initial value problem

$$\begin{cases} 2y'' + 8y' + 6y = 0 \\ y(0) = 1 \\ y'(0) = 0. \end{cases}$$

Problem 3 [10 points]. Find the general solution to the following differential equations.

(a) [2 points] $y'' + y = 0$.

(b) [4 points] $y'' + y = 2e^t$.

(c) [4 points] $y'' + y = 2\cos t$.

Problem 4 [10 points total]. A 2 kg mass is on a large spring with spring constant $k = 8$ N/m and no forcing. Let $y(t)$ denote the motion of the mass in units of meters and seconds.

(a) [7 points] Suppose there is no damping with $y(0) = 4$ m and $y'(0) = -6$ m/s. Solve for the motion $y(t)$ and identify the amplitude of this simple harmonic motion.

(b) [3 points] Now suppose there is damping with damping coefficient $\gamma = 10$ kg/s. Is this under-damped, over-damped, or critically-damped?

Problem 5 [10 points]. A mass m is attached to a spring oriented horizontally with spring constant 6 N/m and forcing function $f(t) = \cos(2t)$, so the motion $y(t)$ satisfies

$$my'' + 6y = \cos(2t).$$

If the spring stretches more than 1 m beyond equilibrium, it might break. If the mass starts at rest in the equilibrium position, what range of values for m risks breaking the spring?

Hint/clarification: when $y(t) = A\sin(\omega_1 t)\sin(\omega_2 t)$, the spring might break if $|A| > 1$.