MATH 207A MIDTERM 1

INTRO TO DIFFERENTIAL EQUATIONS

JULY 10, 2024

NAME: _____

Exam Guidelines.

- Write your name above.
- Remain on this title sheet until you are instructed to begin the exam.
- The time given for this exam is 60 minutes.
- There are 5 questions and each question is worth 10 points.
- You may not use any electronic device except for a non-graphing, non-programmable calculator (such as a TI30 or equivalent). A calculator is not necessary for this exam.
- For full credit, show your work for each problem directly below the problem statement and on the following page, and clearly mark your final answer. For questions with multiple parts, clearly mark which part you are working on.
- Let me know if you have a question about what a problem is asking. If you are taking this exam at a proctoring center, the proctor can direct questions to seanhr@uw.edu.

Good luck!

Problem 1 [10 points total]. A salmon population $P(t)$ is modeled by the differential equation

$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{2}\right) + 3.$$

(a) [6 points] Identify all equilibrium solutions of this differential equation and classify each equilibrium solution as "stable", "unstable", or "semistable".

(b) [4 points] Suppose we have the initial condition $P(0) = 0$. Use Euler's method with stepsize 1 to approximate the value of $P(2)$ (so use 2 steps).

Problem 2 [10 points]. Find the particular solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{ty^2 + t}{y + t^2 y} \\ y(1) = 1. \end{cases}$$

Problem 3 [10 points]. Find the general solution to the differential equation

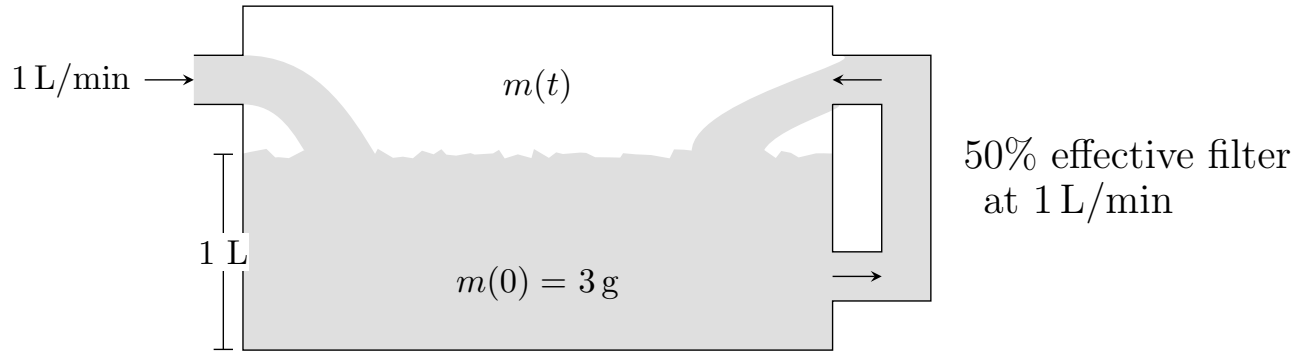
$$\sin(t)y' + \cos(t)y = t\sin(t).$$

Problem 4 [10 points total]. A large tank begins with 1 liter of salt water containing 3 grams of salt. The water in the tank is circulated through a filter at a rate of 1 liter per minute, and this filter removes 50% of the salt in the water that passes through. Additionally, fresh water is pumped into the tank at a rate of 1 liter per minute.

(a) [3 points] Write down the initial value problem that models the mass $m(t)$ of salt in the tank using units of grams and minutes.

(b) [5 points] Solve for the mass $m(t)$ of salt in the tank as a function of time.

(c) [2 points] Rewrite the differential equation you found for (a) using units of kilograms and hours. (1 kilogram is 1000 grams and 1 hour is 60 minutes).



Problem 5 [10 points]. You do not remember the acceleration of gravity g of earth, so you try to approximate g by dropping a cotton ball from a height of 3.5 meters. You observe it takes 1 second for the cotton ball to hit the ground. You know that this cotton ball has a drag coefficient of 1 s^{-1}, so you assume the velocity $v(t)$ of the cotton ball satisfies

$$\frac{dv}{dt} = g - v.$$

You also remember the distance $x(t)$ the cotton ball falls is related to $v(t)$ by

$$\frac{dx}{dt} = v(t).$$

Solve for the value of g that aligns with your observations.